Australian Mathematics Learning in an International Context¹

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The OECD's Programme for International Student Assessment (PISA) measures the achievements of 15-year-olds on a three-yearly cycle. In 2000, reading was the major domain of assessment and mathematics and science were minor domains. In 2003, mathematics was the major domain and reading, science and problem solving were minor domains. In 2006, science will be the major domain. The paper will review the mathematics results from PISA 2000, with the results of additional analyses of the relationship between social background and mathematics achievement across countries, including separate analyses by gender for Australia and some other countries. Results from PISA 2003 will be published on 7 December 2004. The paper will present details of the assessment framework used for testing mathematics achievement in PISA 2003.

Establishment of PISA

The Organisation for Economic Co-operation and Development (OECD) develops a wide range of education statistics and indicators to facilitate quantitative, international comparisons of education systems. These are currently published annually in *Education at a Glance* and are also available on the OECD website (www.oecd.org). Data are provided for the 30 OECD Member countries² and, in recent years, for up to 20 non-Members³. Initially, the focus on outcomes in these statistics and indicators was restricted to completion rates of various levels of education and a range of labour market outcomes.

In 1997, OECD established the Programme for International Student Assessment (PISA) to assess directly levels of student learning in a range of domains. The programme is governed by the participating OECD countries and they have collectively determined the scope and shape of the programme. The first data collection was in 2000, with the first international report published in 2001 (OECD, 2001). The programme is scheduled to continue on a three-yearly cycle. Data were collected in 2003, with the first international report due to be published in December 2004. Planning is now well underway for PISA 2006.

In PISA 2000, the major domain of assessment was reading literacy, with mathematical and scientific literacy as minor domains. PISA 2003 assessed those same three domains, with mathematical literacy as the major domain, and with problem solving added as a fourth. The population assessed is 15-year-olds in school.

There were 43 countries involved in PISA 2000: 28 of the then 29 OECD Members, 4 non-Members that collected data in 2000 and 11 non-Members that used the same instruments in 2002. The results for all 43 countries are published in OECD and UNESCO (2003). In PISA 2003, 41 countries were involved: all 30 current OECD Members and 11

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Australasia, *Mathematics Education for the Third Millennium: Towards 2010*, Townsville, 27-30 June 2004. ² The OECD grew out of the Marshall Plan under which Europe was reconstructed following World War II.

There were 20 Members when OECD was established in 1960. Australia became a Member in 1971.

⁵ Data for these countries are collected through the World Education Indicators programme conducted jointly by UNESCO and OECD, with the support of the World Bank.

non-Members. For PISA 2006, all 30 OECD Members will be involved and 25-30 non-Members.

Assessment in PISA 2000

Earlier quantitative, international studies of educational achievement conducted by the International Association for the Evaluation of Educational Achievement (IAEA) established their assessment frameworks by examining the specifics of the curricula in the participating countries to identify common components^{4.} In PISA, the countries chose to focus instead on students' capacity to use the knowledge and skills they had acquired. Focusing on the curriculum involves looking back to see if students have learned what was intended. Focusing on their capacity to use the knowledge and skills they have acquired involves looking ahead and seeking to judge their preparedness for adult life. This does not imply that all learning required for adult life occurs in school or has been developed by 15-years-of-age; only that important foundations should have been laid by then and that assessing them is an important way in which to judge the quality of the outputs of schooling.

Definition of Literacy

This capacity to use competencies is taken to be 'literacy'. The PISA concept of 'literacy' thus covers a range of competencies. Further, it is not conceptualised in terms of a literate/illiterate dichotomy but rather as a continuum of increasing literacy. PISA is deliberately limited in scope – assessing only reading, mathematical and scientific literacy – but these are important foundations for further learning and for effective participation in adult life.

Mathematical literacy is defined in PISA as:

The capacity to identify, understand, and engage in mathematics and to make well-founded judgements about the role that mathematics plays in an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen (OECD, 2001, p. 22).

It is seen as depending on:

Mathematical knowledge and skills, including basic number facts and operations, fundamental ideas about space and shape, notions of uncertainty, growth and change and skills in working with money and measurements.

The ability to think and work mathematically, including modelling, problem solving, following and evaluating mathematical arguments, posing mathematical problems, choosing ways of representing mathematical situations, expressing oneself on matters with a mathematical content, and knowing the extent and limits of mathematical concepts.

The ability to apply the knowledge and skills in a wide variety of personal, social and work contexts (OECD, 2003, p. 11).

Dimensions for Assessment

The assessment of literacy in each of the three domains is organised on three dimensions: process skills; knowledge and understanding; and the context of application.

⁴ For example, the Third International Mathematics and Science Study (TIMSS) conducted in 1994-95 and its predecessor studies of mathematics and science achievement, commencing in the 1960s.

Process skills:

Mathematical competencies, e.g., modelling, problem solving; divided into three classes: i) carrying out procedures; ii) making connection and integration for problem solving; and iii) mathematisation, mathematical thinking and generalisation (OECD, 2000, p.13).

Content:

Primarily mathematical 'big ideas': chance, change and growth, space and shape, reasoning, uncertainty and dependency relationships. In PISA 2000, when mathematical literacy was a minor domain, assessment was limited to change and growth, and space and shape (OECD, 2000, p. 14).

Context:

Problems that affect individuals, communities or the whole world (OECD, 2000, p. 16).

Assessment tasks for mathematical literacy

Some tasks were assessed through multiple-choice questions. These were typically those involving simpler mathematical processes. Open-ended questions were preferred. Students were required to show the steps taken and to explain how they reached their answer. The distribution of item types is shown in Table 1.

Categories	Number of Items	Multiple-choice	Closed constructed- response	Open constructed- response
Overarching concept ("big idea")				
Growth and change	18	6	9	3
Space and shape	14	5	9	
Total	32	11	18	3
Competency class				
Class 1	10	4	6	
Class 2	20	7	11	2
Class 3	2		1	1
Total	32	11	18	3
Mathematical content strands				
Algebra	5		4	1
Functions	5	4		1
Geometry	8	3	5	
Measurement	7	3	4	
Number	1		1	
Statistics	6	1	4	1
Total	32	11	18	3
Context				
Community	4		2	2
Educational	6	2	3	1
Occupational	3	1	2	
Personal	12	6	6	
Scientific	7	2	5	
Total	32	11	18	3

Table 1Distribution of MathematicsIitems in PISA 2000

(Source: Adams & Wu, 2002, PISA 2000 Technical report, Paris, OECD, pp. 28-29)

Sample tasks, drawn from items with adequate field trial data but not included in the PISA 2000 assessments, were published in 2000 to illustrate PISA assessments (OECD, 2000). A larger set was published after the release of the PISA 2000 results, drawing on material actually used in PISA 2000 but leaving enough unpublished to provide for use in

subsequent cycles in order to link the scales and to permit reporting on a common scale over time (OECD, 2002). Two examples are provided in Figures 1 and 2.



Figure 1. PISA 2000 mathematics sample item – a.



Figure 2. PISA 2000 mathematics sample item – b.

(Source: OECD (2002). Sample tasks from PISA 2000, pp. 95-96)

Performance was expressed on a single scale, unlike in the major domain of reading, for which three sub-scales were developed in addition to an overall reading literacy scale. The scale was constructed with an OECD average of 500 and a standard deviation of 100, with the consequence that about two-thirds of the 15-year-olds across the OECD countries scored between 400 and 600.

Calibration of the tasks on the scale permits a substantive interpretation of performances in different score ranges. A fuller definition is provided in OECD (2003, pp. 87-88), but the descriptions of some key levels on the scale are:

Towards the top end, around 750 points:

Students can interpret and formulate problems in terms of mathematics, identify a suitable way of finding a solution, and can negotiate a number of processing steps and use generalisation and argumentation to explain results.

Just above the middle of the scale, around 570 points:

Students can interpret, link and integrate different representations of a problem or different pieces of information; and/or use and manipulate a given model, often involving algebra or other symbolic representations but typically working with given strategies, models or propositions.

<u>At the lower end of the scale, around 380 points</u>: Students can usually complete only a single processing step consisting of reproducing basic mathematical facts or processes, or applying simple computational skills, using information from diagrammatic or text material that is familiar and straightforward and in which a mathematical formulation is provided or readily apparent.

Sampling and Data Collection

PISA uses an age-based, not a grade-based, definition of its population: all 15-yearolds in school. This avoids the problem for international comparisons created by differences in the age of commencement of schooling and in the use of retention in grade (or grade repetition) for students whose progress has been judged to be inadequate. The results from the Progress in International Reading Literacy Study (PIRLS) 2000 illustrate a key problem of a grade-based sample (Mullis, et al., 2003). As Figure 3 shows, there is a strong relationship between the mean age of students tested in countries and the countries' mean performance levels. The correlation, if Turkey is excluded as an outlier, is 0.59.

In the first round of PISA 2000, data were obtained for more than 250 000 students, representing almost 17 million 15-year-olds in school in the 32 countries. Full details on the sampling procedures and response rates for schools and students are provided in OECD (2001), Annex A3 and in Adams and Wu (2002).

Individual students answered paper and pencil tests lasting two hours. A total of seven hours of assessment items were included, with different students taking different combinations. There were 141 reading items organised into nine clusters, each with an estimated administration time of 30 minutes; 32 mathematics items organised into four 15-minute clusters; and 35 science items organised into four 15-minute clusters. These were grouped into nine test booklets in the manner shown in Figure 4.



Figure 3. Relationship between mean age of students tested and mean performance by countries with grade-based sample of students. (Source: Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., & Kennedy, A.M. (2003). *PIRLS 2001 International Report*, Chestnut Hill, MA: Boston College)

Booklet	Block 1	Block 2	Block 3	Block 4
1	\mathbf{R}_1	R_2	R_4	M_4, M_1
2	R_2	R_3	R_5	S_{1}, S_{2}
3	R_3	R_4	R_6	M_3, M_4
4	R_4	R_5	\mathbf{R}_7	S_1, S_4
5	R_5	R_6	\mathbf{R}_1	M_2, M_3
6	R_6	R_7	R_2	S_{2}, S_{3}
7	R_7	R_1	R_3	R_8
8	M_4, M_1	S_{1}, S_{3}	R_8	R ₉
9	S_4, S_2	M_1, M_3	R ₉	R_8

NOTE: R_x is a 30-minute cluster of reading assessments

M_Y is a 15-minute cluster of mathematics assessments

 S_z is a 15-minute cluster of science assessments

(Adams, R., & Wu, M. (Eds) (2002) PISA Technical report. Paris: OECD, p. 23)

Figure 4. Design of test booklets for PISA 2000

Results in Reading Literacy in PISA 2000

Quality of Achievement

The international report on the results of PISA 2000 concentrates primarily on reading as the major domain of assessment. The extent of the reading assessment materials permitted the construction of three subscales: retrieving information, interpreting texts and reflection and evaluation. It also permitted the definition of five levels of achievement on the overall reading literacy scale and thus a somewhat detailed comparison of the distributions of student achievements across countries.

The mean results on the combined reading literacy scale for 27 of the 28 OECD countries⁵ that participated in PISA 2000 are shown in Figure 5. Significance tests of the differences among these means, using multiple comparisons with Bonferroni adjustment, are given in OECD (2001, p. 53). A more appropriate, and less conservative, test would focus not on all possible multiple comparisons but only those for a particular country. That approach was used in the Executive Summary of the international report (OECD, undated, p. 8).

In reading literacy, as Figure 5 shows, Australia is a high-average performer. Among the 27 participating OECD countries, Australian ranks 2nd, along with six other countries. Only Finland performed significantly better. Many others, including the US, France, Germany and all the Scandinavian countries, performed significantly worse on average.

⁵ The Netherlands was excluded from the analyses because its response rate was too low to ensure comparability of its data -27% of schools on the original list and 55% after attempts to replace those that had declined with corresponding schools from a reserve list, compared with the target of 85% school response rate. By comparison, Australia achieved 81% from the original list of schools and 84% after replacement.



Figure 5. Mean results on PISA reading literacy.

(Source: OECD (2001) Knowledge and skills for life: First results from PISA 2000, Fig. 2.4, p. 53)

Equity of Achievement

PISA also provides important information on the equity of the outcomes of educational systems

In addition to completing two hours of assessment, the 15-year-olds involved in PISA completed a 30-minute background questionnaire that collected information important for the interpretation and analysis of the results. Students were asked about characteristics, such as gender, economic and social background, and activities at home and school. (As part of an international option taken up by most countries, students also reported on their attitudes towards learning, familiarity with computers and, under the heading "self-regulated learning", aspects of strategies for managing and monitoring their own learning.)

The information on economic and social background – parents' education and occupation, cultural artefacts in the home – permitted the construction of an index of social background that is comparable across countries.

The relationship between social background and achievement is quite strong, as shown by the distribution of results for the 265,000 15-year-olds on both variables in Figure 6. (Each dot in that figure represents 20,000 students.) The correlation is relatively high (around 0.45 for the OECD as a whole) and the slope of the regression line for the OECD as a whole is quite steep, indicating that increased social advantage, in general, pays off with considerable increase in educational performance. While the general trend is strong, there are many individual exceptions – socially advantaged individuals who do not perform well and students from disadvantaged backgrounds who perform well.

This result has been long established in many national contexts and it can lead to a counsel of despair. Education can be seen as impotent in the face of strong home influences and unable to make a difference or it can be seen as actively playing a socially reproductive role in ensuring that educational advantage is generally conferred where social advantage already exists.

International comparisons, however, show that the strength of the relationship varies across countries. The regression lines for six countries are shown in Figure 6. Each line is drawn from the 5th to the 95th percentile for students in the country. The lines for Australia, the UK, the US and Germany are all significantly steeper than that for the OECD as a whole. Those for Finland and Korea are significantly less steep than that for the OECD as a whole (OECD, 2000, pp. 184-196 and Table 8.1, p. 308). The line for Korea is not linear, indicating that differences in social advantage are associated with smaller differences in average achievement levels as social advantage increases.

The regression lines tend to converge at higher levels of social advantage, though not entirely. Certainly, the differences in average achievement levels between the countries are much greater at lower levels on the social background scale.

Pursuing Quality and Equity Together

On the basis of its average performance in Figure 5, Australian education can be said to be 'high quality' in reading literacy. On the basis of the steepness of its gradient in Figure 6, Australian education can be said to be 'low equity'.



Figure 6. Regressions of reading literacy on social background.

(Source: OECD (2001) Knowledge and skills for life: First results from PISA 2000, Appendix B1, Table 8.1, p. 30)



Figure 7. Relationship between mean and influence of social background.

(Source: OECD (2001) Knowledge and skills for life: First results from PISA 2000, Appendix B1, Table 2.3a, p. 253)

The picture for all 27 OECD countries involved is shown in Figure 7. In this figure, the vertical axis is achievement on the overall reading literacy scale. The horizontal axis is a measure of system equity. It is the difference between the gradient of the regression line for reading achievement on social background for OECD as a whole and that for the country. Countries with steeper gradients than that for the OECD thus have a negative score and are plotted to the left. Countries with gradients less steep than that for the OECD have positive scores and are plotted to the right. The figure is divided into four segments by the vertical and horizontal lines representing the positions for the OECD as a whole on the two axes.

Countries in the top-right quadrant of Figure 7 can be described as 'high-quality, highequity', those in the top-left, Australia among them, as 'high-quality, low-equity', those in the bottom-right as 'low-quality, high-equity' and those in the bottom-left as 'low-quality, low-equity'. Perhaps the most significant feature of this figure is that the top-right quadrant is not empty. There are countries that can achieve quality and equity together. The pursuit of equity need not be at the expense of quality.

Much attention is being given to how these variations across countries in the relationship between social background and achievement might be created. One provisional indication is presented in Figure 8 where countries that differ rather markedly in the degree of stratification of their education systems are highlighted. Countries with a high degree of stratification are shown with names in white on a black background. In Germany, for example, students are separated into schools of different types (academic, vocational) from as early as 11-years-of-age. Some neighbouring countries like Hungary, the Czech Republic and Luxembourg have similar practices. Countries with a low degree of stratification are shown on a grey background with names in black. These countries have essentially comprehensive systems, at least for students up to 15-years-of-age.

There is a clear tendency for those countries that maintain comprehensive systems to be among the higher achievers but no clear tendency to be high or low equity. There is a clear tendency for those countries that stratify their lower secondary school systems to be among the lower average achievers and a tendency for them also to be low-equity.

Results in Mathematical Literacy in PISA 2000

Because reading literacy was the major domain for assessment in PISA 2000, most of the more detailed analyses of the data have focused on it. PISA 2003 will provide equivalent detail for mathematical literacy but PISA 2000 does provide some evidence for that domain.

Quality of Achievement

In mathematical literacy, Australia is also a high average performer, ranking 3rd along with five other countries, as shown in Figure 9. Both Japan and Korea perform significantly better and these two countries performed as well in reading as did Australia. (PISA 2000 also showed Australia to be high-quality in science literacy – ranking 4th with four others with only Korea, Japan and Finland performing significantly better.)



Figure 8. Relationship between mean and influence of social background.

(Source: OECD (2001) Knowledge and skills for life: First results from PISA 2000, Appendix B1, Table 2.3a, p. 253)



Figure 9. Mean results on PISA mathematical literacy.

(Source: OECD (2001) Knowledge and skills for life: First results from PISA 2000, Figure 3.2, p. 79)



Figure 10. Mean gender differences in reading and mathematical literacy.

(Source: OECD (2001) Knowledge and skills for life, Appendix B1, Table 5.1a, p. 276

Equity of Achievement

There are marked gender differences in achievement, as shown in Figure 10. Girls outperform boys in every country in reading literacy at 15-years-of-age. The difference of 34 points on the PISA reading literacy scale between the means for girls and boys in Australia is the 11th largest difference among the 27 OECD countries.

In mathematical literacy, boys outperform girls in all but Iceland and New Zealand. The difference of 12 points on the PISA reading literacy scale between the means for boys and girls in Australia is also the 11th largest difference among the 27 OECD countries.

The relationship between performance in mathematical literacy and social background is relatively strong, as it is for reading literacy. The relationship across the OECD is shown in Figure 11 and the separate relationships for males and females are shown in Figure 12. The best fitting regression lines are slightly curvilinear in all cases, with increasing social advantage being associated with slightly smaller increases in mean mathematical literacy at higher levels of social advantage.



Figure 11. Regressions of mathematical literacy on social background in OECD as a whole.



Figure 12. Regressions of mathematical literacy on social background in OECD by gender.

The relationship for Australia compared with the OECD overall relationship is shown in Figure 13. In reading literacy, the slope of the line for Australia was significantly steeper for Australia than for the OECD overall. In mathematical literacy, they are essentially the same, though the line for Australia is slightly curvilinear in the opposite direction, with additional social advantage being associated with smaller increments in mean mathematical literacy level at lower levels of social advantage and larger increments in mean mathematical literacy level at higher levels of social advantage. The relationships for boys and girls in Australia are essentially the same, as shown in Figure 14. The line for boys is higher, of course, reflecting their higher mean performance but the lines are essentially parallel.



Figure 13. Regressions of mathematical literacy on social background, OECD and Australia.



Figure 14. Regressions of mathematical literacy on social background in Australia by gender.

Comparisons between Australia and three other high performing countries are shown in Figure 15. They are Finland and Canada, both with mean mathematical literacy performances not significantly different from Australia, as shown in Figure 9. The third country, Hong Kong - China, was one of those that used the PISA 2000 assessments in 2002 and for which results are published, together with those of the countries that tested in

2000, in OECD and UNESCO (2003). Hong Kong - China has the highest mean performance in mathematical literacy, and significantly higher than that of Australia.



Figure 15. Regressions of mathematical literacy on social background in Australia and others.

Of the four countries, Australia is the only one in which the best fitting line is concave from above. The more detailed analyses of reading literacy data and social background shows that some countries manage to ameliorate the effects of social background on educational achievement. Australia was not among them. In mathematical literacy, the Australian education system also does less to ameliorate the effects of social background than Finland and Canada. Furthermore, the benefits of an advantaged social background seem to pay off even more in Australia for the most socially advantaged students.

There is a tendency in Australia to dismiss attempts to redress inequities in education by declaring that they will result in 'dumbing down'. The PISA 2000 evidence that some countries achieve high quality and at the same time more equitable results than Australia makes it clear that 'dumbing down' is no necessary consequence of the pursuit of equity. 'Levelling up' is clearly an alternative.

Australia needs to examine carefully the sources of inequity in student performance in its education system to determine where policy intervention might most effectively be made to improve the equity of outcomes without sacrificing policy. This should involve analysis of differences between the public and private sectors, between urban and rural environments and between the States and Territories.

Assessment of Mathematics in PISA 2003

PISA 2003 will provide more detailed information on levels of mathematical literacy in all 30 OECD countries and in 11 others when the results are published in December 2004. The framework for the PISA 2003 mathematical literacy assessments is essentially the same as that for PISA 2000, except in one important respect. While 'process skills' and 'context' are still defined in the same way, 'content' has been broadened because of the additional assessment time available.

For PISA 2000, the content framework was defined in terms of six 'big ideas': chance, change and growth, space and shape, reasoning, uncertainty and dependency relationships. The PISA 2000 assessments, however, were limited to change and growth, and space and shape (OECD, 2000, p.14).

For PISA 2003, four 'overarching ideas' were defined – quantity, space and shape, change and relationships, and uncertainty – and all were assessed. They are defined in OECD (2003), pp.35-37.

An important methodological issue being addressed in the analysis of the PISA 2003 data is how well the mathematical literacy scales from assessment as a minor domain and assessment as a major domain can be linked for the study of growth. The answer to that question will be available in December, together with a large new database on mathematical performance that will be available on the same day for others to begin using.

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